Dr. Kamlesh Kumar
Asst. Prof. (Guest Faculty)
Deptt. of mathematics
Maharaja College,
V.K.S.U, Ara

Date
22/04/21

M.Sc. Sem-II, Paper-VI

Complex Analysis (Continue)

Polar representation of complex numbers

Consider the point $z = x + iy$ in the complex plane. This point has polar co-ordinates (r, θ), where $x = r\cos\theta$ and $y = r\sin\theta$. Thus $z = r(\cos\theta + i\sin\theta)$. Clearly $r = |z| = \sqrt{x^2+y^2}$, which is magnitude of the complex number and θ (undefined if $z = 0$) is the angle between the positive real axis and the line segment from 0 to z and is called the argument of z, denoted by $\theta = \arg z$.

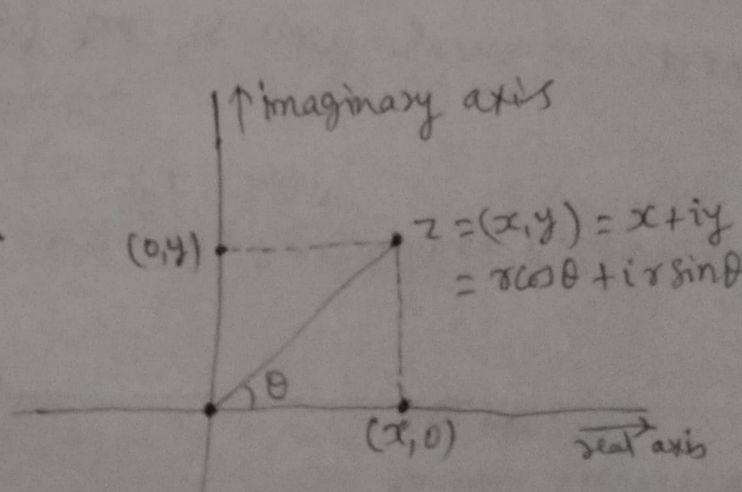
We note that the value of argument of z is not unique. If $\theta = \arg z$, then $\theta + 2\pi n$, where n is an integer is also $\arg z$. The value of $\arg z$ that lies in the range $-\pi < \theta \leq \pi$ is called the principal value of $\arg z$.

If z_1, z_2 are any two non-zero complex numbers then

① $\arg z_1 = -\arg \bar{z}_1$

② $\arg z_1 z_2 = \arg z_1 + \arg z_2$

③ $\arg\left[\frac{z_1}{z_2}\right] = \arg z_1 - \arg z_2$.



We shall simply state

De Moivre's Theorem:- For any real number n, $\cos n\theta + i\sin n\theta$ is one of the values of $(\cos\theta + i\sin\theta)^n$.

nth Roots of Complex Numbers.

Let $z = r(\cos\theta + i\sin\theta)$ be a non-zero complex number, the $w = \rho(\cos\varphi + i\sin\varphi)$ is nth root of z if $w^n = z$, where n is a positive integer.

Therefore, $\rho^n(\cos\varphi + i\sin\varphi)^n = r(\cos\theta + i\sin\theta)$

$$\rho^n(\cos n\varphi + i\sin n\varphi) = r(\cos\theta + i\sin\theta)$$

$$\rho^n = r \text{ and } n\varphi = \theta + 2K\pi, \text{ where } K \text{ is an integer.}$$

However, only the value of $K = 0, 1, 2, ---, (n-1)$ will give distinct value of w. Hence z has n distinct nth roots and they are given by

$$w = r^{\frac{1}{n}}\left[\cos\left(\frac{\theta + 2K\pi}{n}\right) + i\sin\left(\frac{\theta + 2K\pi}{n}\right)\right] \text{ where } K = 0, 1, 2, \cdots, (n-1).$$

Some Topological aspects

A metric space with respect to usual metric $d(z, \zeta) = |z - \zeta|$.

By an open disc, we mean the set $\{z : |z-a| < \epsilon\}$ and is denoted by $B(a;\epsilon)$. And by closed disc, we mean, $\{z : |z-a| \leq \epsilon\}$ and is denoted by $\bar{B}(a;\epsilon)$. Further an annulus is defined as the set $\{z : r < |z-a| < R\}$ and is denoted by $\text{ann}(a; r, R)$.

The punctured disk of radius ϵ centered at a is defined by,

$B(a;\epsilon) - \{0\} = \{z : 0 < |z-a| \leq \epsilon\}$.